

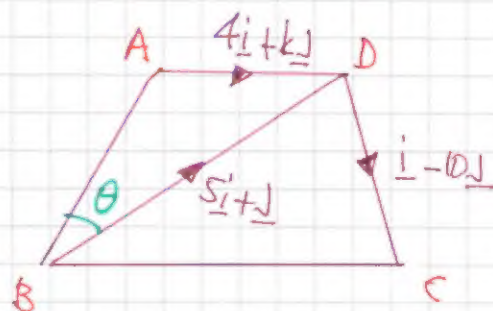
1YG-B - MPI PAPER P - QUESTION 1

a) LOOKING AT THE DIAGRAM

$$\Rightarrow \vec{BC} = \vec{BD} + \vec{DC}$$

$$\Rightarrow \vec{BC} = (5\hat{i} + \hat{j}) + (1 - 10\hat{j})$$

$$\Rightarrow \vec{BC} = 6\hat{i} - 9\hat{j}$$



AS AD IS PARALLEL TO BC, THEIR VECTOR COMPONENTS MUST BE IN PROPORTION

$$\Rightarrow \frac{4}{k} = \frac{6}{-9}$$

$$\Rightarrow 6k = -36$$

$$\Rightarrow k = -6$$

AS REQUIRED

b) FIRST FIND \vec{AB}

$$\vec{AB} = \vec{AD} + \vec{DB} = (4\hat{i} - 6\hat{j}) - (5\hat{i} + \hat{j}) = -\hat{i} - 7\hat{j}$$

NEXT THE LENGTH OF \vec{AB}

$$|\vec{AB}| = |-\hat{i} - 7\hat{j}| = \sqrt{(-1)^2 + (-7)^2} = \sqrt{50} = 5\sqrt{2}$$

c) BY THE COSINE RULE ON $\triangle ABD$

$$|\vec{AD}| = |4\hat{i} - 6\hat{j}| = \sqrt{4^2 + (-6)^2} = \sqrt{52}$$

$$|\vec{BD}| = |5\hat{i} + \hat{j}| = \sqrt{5^2 + 1^2} = \sqrt{26}$$

$$\cos \theta = \frac{|\vec{AB}|^2 + |\vec{BD}|^2 - |\vec{AD}|^2}{2|\vec{AB}||\vec{BD}|} = \frac{50 + 26 - 52}{2 \times 5\sqrt{2} \sqrt{26}} = 0.33282...$$

$$\therefore \theta \approx 70.6^\circ$$

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IYGB - MPI PAPER 2 P - QUESTION 2

THE EQUATION IS ALREADY FACTORIZED SO WE SOLVE DIRECTLY FOR EACH OF THE TWO FACTORS

$$\Rightarrow (\sqrt{3} - 2\sin 3\alpha)(\sqrt{3} + 2\cos 3\alpha) = 0$$

$$\Rightarrow \sqrt{3} - 2\sin 3\alpha = 0$$

$$\Rightarrow \sqrt{3} = 2\sin 3\alpha$$

$$\Rightarrow \sin 3\alpha = \frac{\sqrt{3}}{2}$$

$$\arcsin\left(\frac{\sqrt{3}}{2}\right) = 60^\circ$$

$$\Rightarrow \begin{cases} 3\alpha = 60^\circ \pm 360^\circ n \\ 3\alpha = 120^\circ \pm 360^\circ n \end{cases} \quad n=0,1,2,3,\dots$$

$$\Rightarrow \begin{cases} \alpha = 20^\circ \pm 120^\circ n \\ \alpha = 40^\circ \pm 120^\circ n \end{cases}$$

$$\Rightarrow \sqrt{3} + 2\cos 3\alpha = 0$$

$$\Rightarrow 2\cos 3\alpha = -\sqrt{3}$$

$$\Rightarrow \cos 3\alpha = -\frac{\sqrt{3}}{2}$$

$$\arccos\left(-\frac{\sqrt{3}}{2}\right) = 150^\circ$$

$$\Rightarrow \begin{cases} 3\alpha = 150^\circ \pm 360^\circ n \\ 3\alpha = 210^\circ \pm 360^\circ n \end{cases} \quad n=0,1,2,3,\dots$$

$$\Rightarrow \begin{cases} \alpha = 50^\circ \pm 120^\circ n \\ \alpha = 70^\circ \pm 120^\circ n \end{cases}$$

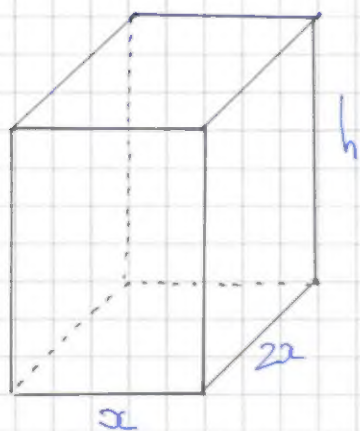
$$\begin{aligned} \alpha_1 &= 20^\circ \\ \alpha_2 &= 140^\circ \\ \alpha_3 &= 40^\circ \\ \alpha_4 &= 160^\circ \\ \alpha_5 &= 50^\circ \\ \alpha_6 &= 170^\circ \\ \alpha_7 &= 70^\circ \end{aligned}$$

OR $\alpha = 20^\circ, 40^\circ, 50^\circ, 70^\circ, 140^\circ, 160^\circ, 170^\circ$

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YGB - MPI PAGE P - QUESTION 3

a)



CONSTRAINT $V = 1000 \text{ cm}^3$

$$\Rightarrow V = x(2x)h$$

$$\Rightarrow 1000 = 2x^2h$$

$$\Rightarrow x^2h = 500$$

SURFACE AREA ($A \text{ cm}^2$)

$$\Rightarrow A = 2[2x^2 + xh + 2xh]$$

$$\Rightarrow A = 4x^2 + 6xh$$

$$\Rightarrow A = 4x^2 + \frac{3000}{x}$$

$$xh = \frac{500}{x}$$

$$6xh = \frac{3000}{x}$$

As required

b)

$A = 4x^2 + 3000x^{-1}$

$$\Rightarrow \frac{dA}{dx} = 8x - 3000x^{-2}$$

For STATIONARY VALUES $\frac{dA}{dx} = 0$

$$\Rightarrow 8x - \frac{3000}{x^2} = 0$$

$$\Rightarrow 8x = \frac{3000}{x^2}$$

$$\Rightarrow 8x^3 = 3000$$

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LYGB - MPI PAPER P - QUESTION 3

$$\Rightarrow x^3 = 375$$

$$\Rightarrow \underline{x = \sqrt[3]{375} \approx 7.21 \text{ cm}}$$

c) $\underline{A = 4x^2 + \frac{3000}{x}}$

$$\Rightarrow A_{\min} = 4(7.21\dots)^2 + \frac{3000}{7.21\dots}$$

$$\Rightarrow \underline{A_{\min} \approx 624 \text{ cm}^2}$$

To JUSTIFY IT IS A MIN, USE 2ND DERIVATIVE

$$\Rightarrow \frac{dA}{dx} = 8x - 3000x^{-2}$$

$$\Rightarrow \frac{d^2A}{dx^2} = 8 + 6000x^{-3}$$

$$\Rightarrow \frac{d^2A}{dx^2} = 8 + \frac{6000}{x^3}$$

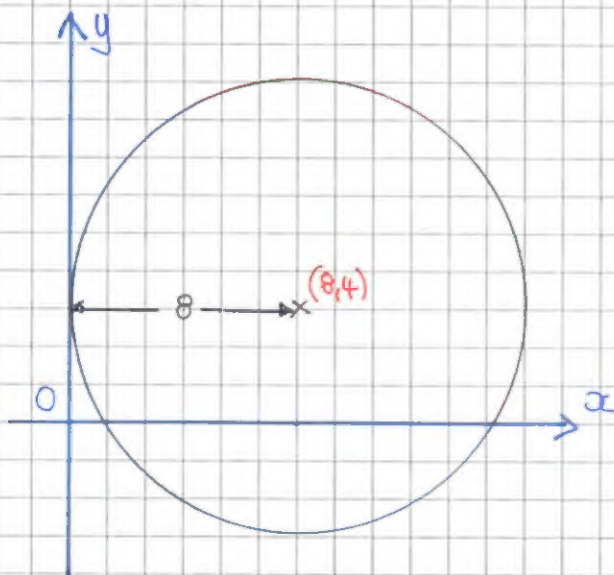
$$\Rightarrow \left. \frac{d^2A}{dx^2} \right|_{x=7.21\dots} = 8 + \frac{6000}{(7.21\dots)^3} = 24 > 0$$

$$x = 7.21\dots$$

INDICATES A MINIMUM

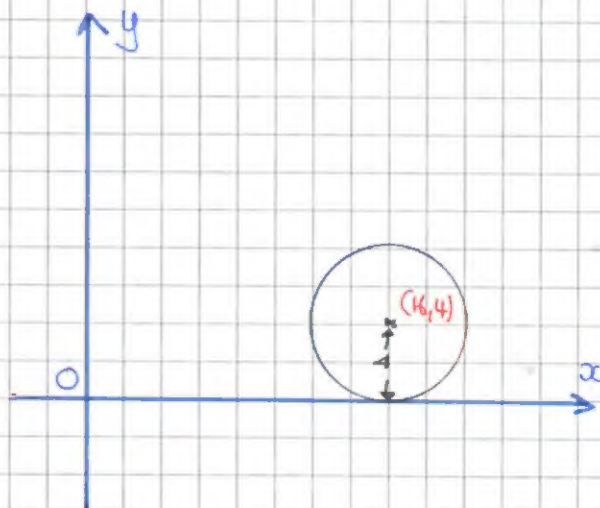
YGB - MPI PAPER P - QUESTION 4

a) DRAWING EACH OF THE CIRCLES



$$\therefore (x-8)^2 + (y-4)^2 = 8^2$$

$$(x-8)^2 + (y-4)^2 = 64$$



$$\therefore (x-16)^2 + (y-4)^2 = 4^2$$

$$(x-16)^2 + (y-4)^2 = 16$$

b) SOLVING THE EQUATIONS SIMULTANEOUSLY

$$(x-8)^2 + (y-4)^2 = 64$$

$$(x-16)^2 + (y-4)^2 = 16$$

SUBTRACTING EQUATIONS

$$\Rightarrow (x-8)^2 - (x-16)^2 = 48$$

$$\Rightarrow x^2 - 16x + 64 - (x^2 - 32x + 256) = 48$$

$$\Rightarrow x^2 - 16x + 64 - x^2 + 32x - 256 = 48$$

$$\Rightarrow 16x = 240$$

$$\Rightarrow x = 15$$

SUBSTITUTING INTO EITHER EQUATION

$$\Rightarrow (x-16)^2 + (y-4)^2 = 16$$

$$\Rightarrow (15-16)^2 + (y-4)^2 = 16$$

IYGB - MPI PAPER P - QUESTION 4

$$\Rightarrow (-1)^2 + (y-4)^2 = 16$$

$$\Rightarrow 1 + (y-4)^2 = 16$$

$$\Rightarrow (y-4)^2 = 15$$

$$\Rightarrow y-4 = \pm\sqrt{15}$$

$$\Rightarrow y = 4 \pm \sqrt{15}$$

THUS THE REQUIRED CO-ORDINATES IN ANY ORDER ARE

$A(15, 4+\sqrt{15})$ & $B(15, 4-\sqrt{15})$ //

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1YGB - MPI PAPER P - QUESTION 5

$$(1 + kx)^n = 1 + Ax + 264x^2 + 1760x^3 + \dots$$

a) EXPAND $(1 + kx)^n$ UP AND INCLUDING THE TERM IN x^3

$$\begin{aligned}(1 + kx)^n &= 1 + \frac{n}{1}(kx)^1 + \frac{n(n-1)}{1 \times 2}(kx)^2 + \frac{n(n-1)(n-2)}{1 \times 2 \times 3}(kx)^3 + \dots \\ &= 1 + \underline{nkx} + \underline{\frac{1}{2}n(n-1)k^2x^2} + \underline{\frac{1}{6}n(n-1)(n-2)k^3x^3}\end{aligned}$$

COMPARING COEFFICIENTS IN x^2 & x^3

$$\bullet \frac{1}{2}n(n-1)k^2 = 264$$

$$\bullet \frac{1}{6}n(n-1)(n-2)k^3 = 1760$$

$$\frac{1}{3}\left[\frac{1}{2}n(n-1)k^2\right](n-2)k = 1760$$

$$\frac{1}{3} \times 264(n-2)k = 1760$$

$$\underline{(n-2)k = 20}$$

As required

b) NOW WE HAVE

$$\bullet \frac{1}{2}n(n-1)k^2 = 264$$

$$n(n-1)k^2 = 528$$

$$\bullet (n-2)k = 20$$

$$(n-2)^2k^2 = 400$$

DIVIDING THE EQUATIONS SIDE BY SIDE

$$\frac{n(n-1)k^2}{(n-2)^2k^2} = \frac{528}{400}$$

1YGB - UPI PAPER P - QUESTION 5

$$\Rightarrow \frac{n(n-1)}{(n-2)^2} = \frac{33}{25}$$

$$\Rightarrow \frac{n^2 - n}{n^2 - 4n + 4} = \frac{33}{25}$$

$$\Rightarrow 25n^2 - 25n = 33n^2 - 132n + 132$$

$$\Rightarrow 0 = 8n^2 - 107n + 132$$

$$\Rightarrow 0 = (8n - 11)(n - 12)$$

(OR BY THE QUADRATIC FORMULA

$$n = \frac{107 \pm \sqrt{(-107)^2 - 4 \times 8 \times 132}}{2 \times 8}$$

$$n = \frac{107 \pm \sqrt{7225}}{16} =$$

$$n = \frac{107 \pm 85}{16} = \left\langle \begin{array}{l} 12 \\ \frac{11}{8} \end{array} \right\rangle$$

$$\Rightarrow n = \left\langle \begin{array}{l} 12 \\ \frac{11}{8} \end{array} \right\rangle$$

|||WCE USING $(n-2)k = 20$

$$10k = 20$$

$$k = 2$$

FINALLY $A = nk$

$$A = 12 \times 2$$

$$\therefore \underline{A = 24}$$

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1YGB - MPI PART P - QUESTION 6

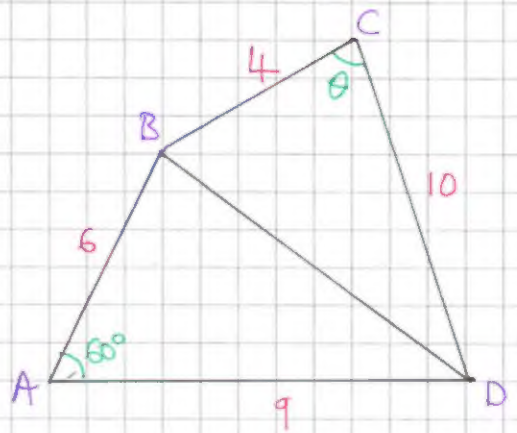
a) BY THE COSINE RULE ON $\triangle ABD$

$$\Rightarrow |BD|^2 = |AB|^2 + |AD|^2 - 2|AB||AD|\cos 60^\circ$$

$$\Rightarrow |BD|^2 = 36 + 81 - 2 \times 6 \times 9 \times \frac{1}{2}$$

$$\Rightarrow |BD|^2 = 63$$

$$\Rightarrow |BD| = \sqrt{63} = 3\sqrt{7} \quad \text{As required}$$



b) BY THE COSINE RULE ON $\triangle BCD$

$$\Rightarrow |BD|^2 = |BC|^2 + |CD|^2 - 2|BC||CD|\cos \theta$$

$$\Rightarrow 63 = 16 + 100 - 2 \times 4 \times 10 \times \cos \theta$$

$$\Rightarrow 80 \cos \theta = 53$$

$$\Rightarrow \cos \theta = \frac{53}{80}$$

$$\Rightarrow \theta \approx 48.5^\circ$$

c) FINDING THE AREA OF EACH OF THE TRIANGLES

$$\text{AREA OF } \triangle ABD = \frac{1}{2} \times 6 \times 9 \times \sin 60^\circ \approx 23.382 \dots$$

$$\text{AREA OF } \triangle BCD = \frac{1}{2} \times 4 \times 10 \times \sin(48.5^\circ) \approx 14.981 \dots$$

$$\therefore \text{TOTAL AREA} = 38.4 \text{ cm}^2 \quad (3 \text{ sf})$$

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IYGB - MPI PAPER P - QUESTION 7

LET THE TWO ROOTS BE α & $\alpha+k$

THEN

$$(x - \alpha)[x - (\alpha + k)] = 0$$

$$x^2 - (\alpha + k)x - \alpha x + \alpha(\alpha + k) = 0$$

$$x^2 - (2\alpha + k)x + (\alpha^2 + k\alpha) = 0$$

THUS $b = -(2\alpha + k)$ & $c = \alpha^2 + \alpha k$

$$\Rightarrow b^2 - 4ac = [-(2\alpha + k)]^2 - 4 \times 1 \times (\alpha^2 + \alpha k)$$

$$= (2\alpha + k)^2 - 4(\alpha^2 + \alpha k)$$

$$= \cancel{4\alpha^2} + \cancel{4\alpha k} + k^2 - \cancel{4\alpha^2} - \cancel{4\alpha k}$$

$$= \underline{k^2}$$

1YGB - MPI PAPER P - QUESTION 8

a) WRITE IN INDEX NOTATION AND DIFFERENTIATE

$$\Rightarrow y = 4\sqrt{x} - 3x - 3$$

$$\Rightarrow y = 4x^{\frac{1}{2}} - 3x - 3$$

$$\Rightarrow \frac{dy}{dx} = 2x^{-\frac{1}{2}} - 3$$

$$\Rightarrow \left. \frac{dy}{dx} \right|_{x=4} = 2 \times 4^{-\frac{1}{2}} - 3 = -2 \leftarrow \text{TANGENT GRADIENT}$$

FIND THE y CO-ORDINATE OF P & THEN THE TANGENT

$$y \Big|_{x=4} = 4\sqrt{4} - 3 \times 4 - 3 = -7$$

$$\therefore y - y_0 = m(x - x_0)$$

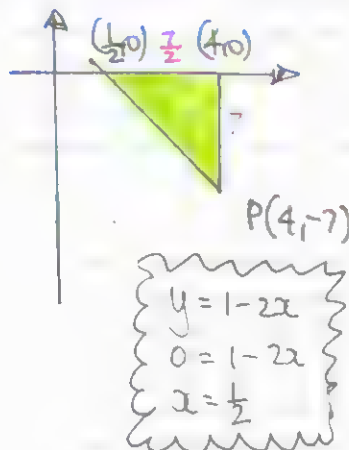
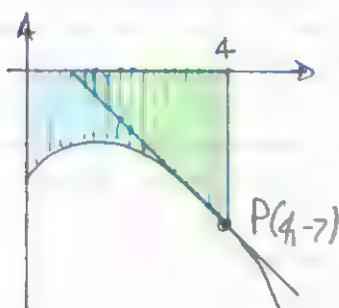
$$y + 7 = -2(x - 4)$$

$$y + 7 = -2x + 8$$

$$y = 1 - 2x$$

* REQUIRED

b) LOOKING AT THE PICTORIAL EQUATION BELOW



AREA ABOVE IS GIVEN BY

$$- \int_0^4 4x^{\frac{1}{2}} - 3x - 3 \, dx$$

$$\text{AREA OF TRIANGLE} = \frac{1}{2} \times 7 \times \frac{7}{2} = \frac{49}{4}$$

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IYGB - MPI PAPER P - QUESTION 8

$$\begin{aligned} &= - \int_0^4 4x^{\frac{1}{2}} - 3x - 3 \, dx = \int_4^0 4x^{\frac{1}{2}} - 3x - 3 \, dx \\ &= \left[\frac{8}{3}x^{\frac{3}{2}} - \frac{3}{2}x^2 - 3x \right]_4^0 = 0 - \left(\frac{64}{3} - 24 - 12 \right) \\ &= \frac{44}{3} \end{aligned}$$

Hence the required area can be found

$$\frac{44}{3} - \frac{49}{4} = \frac{176}{12} - \frac{147}{12} = \frac{29}{12}$$

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IYGB - MPI PAPER P - QUESTION 9

USING THE RULES OF LOGARITHMS

$$\Rightarrow 2\log_2 x + \log_2(x-1) - \log_2(5x+4) = 1$$

$$\Rightarrow \log_2 x^2 + \log_2(x-1) - \log_2(5x+4) = \log_2 2$$

$$\Rightarrow \log_2 \left[\frac{x^2(x-1)}{5x+4} \right] = \log_2 2$$

$$\Rightarrow \frac{x^2(x-1)}{5x+4} = 2$$

$$\Rightarrow \frac{x^3 - x^2}{5x+4} = 2$$

$$\Rightarrow x^3 - x^2 = 10x + 8$$

$$\Rightarrow x^3 - x^2 - 10x - 8 = 0$$

LOOK FOR "OBVIOUS" SOLUTIONS $\pm 1, \pm 2, \pm 4, \pm 8$

$$\bullet x=1 \quad 1-1-10-8 \neq 0$$

$$\bullet x=-1 \quad -1-1+10-8=0 \quad !!$$

$\therefore (x+1)$ IS A FACTOR

BY LONG DIVISION OR MANIPULATIONS

$$\Rightarrow x^2(x+1) - 2x(x+1) - 8(x+1) = 0$$

$$\Rightarrow (x+1)(x^2 - 2x - 8) = 0$$

$$\Rightarrow (x+1)(x+2)(x-4) = 0$$

$$\Rightarrow x = \begin{cases} -1 \\ -2 \\ 4 \end{cases}$$

ONLY $x=4$ IS ACCEPTABLE FOR THE ARGUMENT OF $\log_2 x$

LYGB - MPI PAPER P - QUESTION 10

- a) AS WE REQUIRED TO FIND THE AREA WE SHALL WORK WITH LENGTHS AND PYTHAGORAS TO SHOW $\hat{A}BD = 90^\circ$
(RATHER THAN GRADIENTS)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\begin{aligned} \bullet |AB| &= \sqrt{(-4+3)^2 + (2+2)^2} \\ &= \sqrt{1+16} \\ &= \sqrt{17} \end{aligned}$$

$$\bullet |BD| = \sqrt{(-4-4)^2 + (2-4)^2} = \sqrt{64+4} = \sqrt{68}$$

$$\bullet |AD| = \sqrt{(-3-4)^2 + (-2-4)^2} = \sqrt{49+36} = \sqrt{85}$$

$$\Rightarrow |AB|^2 + |BD|^2 = (\sqrt{17})^2 + (\sqrt{68})^2 = 17 + 68 = 85 = (\sqrt{85})^2 = |AD|^2$$

THE PYTHAGOREAN RELATIONSHIP IS SATISFIED FOR A RIGHT ANGLE AT $\hat{A}BD$

FINALLY THE AREA OF THE PARALLELOGRAM IS THAT OF 2 IDENTICAL TRIANGLES

$$\begin{aligned} \Rightarrow \text{AREA} &= 2 \times \text{AREA OF } \triangle ABD \\ &= 2 \times \frac{1}{2} \times |AB| |BD| \\ &= \sqrt{17} \times \sqrt{68} \\ &= \sqrt{17} \times 2\sqrt{17} \\ &= 34 \end{aligned}$$

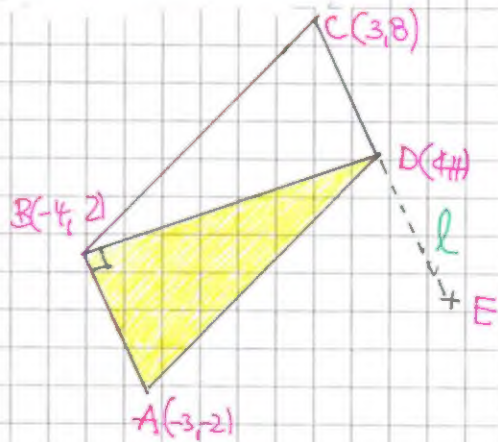
- b) START WITH THE GRADIENT OF CD

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 8}{4 - 3} = \frac{-4}{1} = -4$$

EQUATION OF l IS GIVEN BY

$$y - y_0 = m(x - x_0)$$

$$y - 4 = -4(x - 4) \quad (\text{USED } D(4,4))$$



LYGB - MPI PAPER P - QUESTION 10

WHEN $y=0$ THE EQUATION REDUCES TO

$$0-4 = -4(x-4)$$

$$1 = x-4$$

$$x=5$$

$$\therefore \underline{E(5,0)}$$

c) CONSIDERING MIDPOINTS

$$\text{MIDPOINT OF AD} = \left(\frac{-3+4}{2}, \frac{-2+4}{2} \right) = \left(\frac{1}{2}, 1 \right)$$

$$\text{MIDPOINT OF BE} = \left(\frac{-4+5}{2}, \frac{2+0}{2} \right) = \left(\frac{1}{2}, 1 \right)$$

INDEED THEY BISECT EACH OTHER

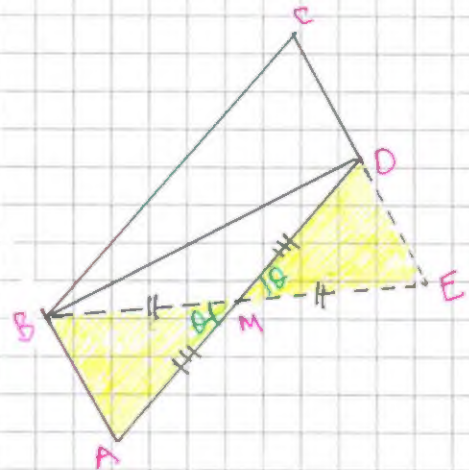
d) LOOKING AT A DETAILED DIAGRAM

$$|AM| = |MD| \quad (\text{part c})$$

$$|BM| = |ME| \quad (\text{part c})$$

$$\hat{BMA} = \hat{DME} \quad (\text{vertically opposite})$$

$$\therefore \triangle ABM \text{ IS CONGRUENT TO } \triangle DME$$



$$\therefore \underline{\text{AREA OF } \triangle EBC = \text{AREA OF } \triangle ABC}$$

REMOVE $\triangle DME$ FROM $\triangle EBC$ & PLACE IN THE POSITION OF $\triangle ABM$,
TO FORM PARALLELOGRAM ABCD

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(YGB - MPI PAPER P - QUESTION 11)

a)

$$\text{LET } y = f(x) = 8^x$$

I)

THW

$$\left(\frac{1}{8}\right)^x = (8^{-1})^x = 8^{-x} = f(-x)$$

\therefore REFLECTION ABOUT THE y AXIS

II)

THW

$$2^x = \left(8^{\frac{1}{3}}\right)^x = 8^{\frac{1}{3}x} = f\left(\frac{1}{3}x\right)$$

\therefore HORIZONTAL STRETCH, BY SCALE FACTOR 3

(OR ENLARGEMENT PARALLEL TO THE x AXIS BY SCALE FACTOR 3)

b)

PROCEED AS FOLLOWS

$$8^{x-1} = 8^x \times 8^{-1} = \frac{1}{8}(8^x) = \frac{1}{8}f(x)$$

\therefore VERTICAL STRETCH BY SCALE FACTOR OF $\frac{1}{8}$

(OR ENLARGEMENT PARALLEL TO THE y AXIS BY SCALE FACTOR $\frac{1}{8}$)